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Statistical Analysis of Core Strength of Deep Cement Mixing Ground

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ABSTRACT Cement-treated ground by deep cement mixing has been used for several geotechnical structures widely. In quality assurance processes of this method, statistical parameters, mean and standard deviation, of unconfined compressive strength of cement-treated soils are used. The mean and standard deviation are normally used to assure the quality of the improved ground. These parameters evaluated from core strength data are the sample statistical parameters, indicating these parameters involve the statistical uncertainty. Thus the evaluation of the statistical uncertainty is needed when assuring the quality of the improved ground precisely. Moreover, the spatial correlation exists in core strength data. The statistical uncertainty emerging in the evaluation of the population statistical parameters is possibly affected by the spatial correlation. This paper presents the statistical analysis of core strength data observed in several deep cement mixing projects. The mean, standard deviation, and autocorrelation distance, were adopted as the statistical parameters of the strength. The type of the probability distribution of the core strength was investigated by the Kolmogoronv-Smirnov (K-S) test. The goodness fit of the normal and log-normal distributions was examined against the core strength data. The autocorrelation distance, which is the parameter representing the characteristic of the spatial correlation, was calculated from the distribution of the core strength using the maximum-likelihood method. The statistical uncertainty of the statistical parameters was evaluated using a Bayesian inference approach. In the Bayesian inference approach, a Markov chain Monte Carlo method was adopted to calculate the realizations of the population statistical parameters. The analysis results indicated the statistical uncertainty included in the statistical parameters is significantly affected by the spatial correlation.

KEYWORDS Deep Cement Mixing; Core Strength; Statistical Analysis; Spatial Variability; Statistical Uncertainty

1 INTRODUCTION

Cement-treated ground by deep cement mixing has been used for several geotechnical structures widely. In quality assurance processes of this method, the unconfined compressive strength q_{uf} of cement-treated soils retrieved from the improved ground, core strength, is measured and its statistical parameters, mean and standard deviation, are adopted to check the strength of the constructed ground. Since the mean and standard deviation of core strength are sample statistical parameters, these values include statistical uncertainty emerging in the evaluation of the population statistical parameters. Moreover, the strength of deep cement mixing ground has a spatial correlation (Namikawa and Koseki 2013). The statistical uncertainty could be affected by the spatial correlation. Thus in the quality assurance procedure, the spatial correlation of the core strength should be considered when using the statistical parameters.

In the evaluation of an overall performance of the cement-treated ground in quality assurance processes, the spatial variability and the statistical uncertainty should be considered properly. The author has proposed the analytical approach for evaluating the overall strength of the cement-treated ground when considering the spatial variability and the statistical uncertainty simultaneously (Namikawa 2021; Namikawa 2022). These previous studies indicated that the both uncertainties are important to evaluate the variability of the predicted overall strength of the cement-treated ground. However, there is little study on the statistical uncertainty including in the statistical parameters observed in real projects.

This paper presents the statistical analysis of the core strength data observed in several deep cement mixing projects. The mean μ_{quf}, standard deviation σ_{quf}, and autocorrelation distance θ_{quf} of q_{uf} of cement treated soils were evaluated as the statistical parameters. θ_{quf} represents the spatial correlation of q_{uf}. The maximum-likelihood method was used to evaluate θ_{quf} from the core strength distributions. The type of the probability distribution of q_{uf} was investigated by the Kolmogoronv-Smirnov (K-S) test. The goodness fit of the normal and log-normal distributions was examined for q_{uf}. Moreover, a Bayesian inference approach was used to evaluate the statistical uncertainty involved in the statistical parameters. In this approach, the posterior distribution of the statistical parameters was inferred from the observed core strength data and the prior distributions of the statistical parameters. A Markov chain Monte Carlo (MCMC) method was used to calculate the realizations of μ_{quf}, σ_{quf}, and θ_{quf}. The analysis results provide quantitatively the statistical uncertainty emerging in the evaluation of μ_{quf}, σ_{quf}, and θ_{quf}.

2 STATISTICAL CHARACTERISTIC OF CORE STRENGTH DATA

2.1 Core Strength Data

The core strength data obtained in 4 deep cement mixing projects (Namikawa and Koseki 2013; Babasaki and Suzuki 1996; Onimaru et al. 2009) were analyzed here. Figure 1 shows the distribution of the core strength data used in this study. Three core boring data were obtained in project A. In this site, since the upper deposit is sand and the lower deposit is clay, the data were separated into the cement-treated sand and clay at each boring.

2.2 Statistical Parameters

The sample statistical parameters of q_{uf} are summarized in Table 1. In project A, in addition to analyzing each core boring data separately, three core boring data are analyzed together. In project B, two core boring data are analyzed separately, because the cement columns were constructed with different cement contents.

The sample mean $s\mu_{\text{quf}}$ of the core strength of cement-treated sands ranges from 5.4 MPa to 10. 5MPa and that of cement-treated clays ranges from 2.8 MPa to 6.1MPa. $s\mu_{\text{quf}}$ of the cement-treated clay is smaller than that of the cement-treated sand. The coefficient of variation sV_{quf} of the core strength of cement-treated sand for a single column ranges from 0.20 to 0.37 and that of cement-treated clay

Table 1. Sample statistical parameters of core strength data

Project	Boring No.	Soil	N_{b}	n	$s\mu_{\text{quf}}$	$s\sigma_{\text{quf}}$	sV_{quf}	$s\theta_{\text{quf}}$
A	A-1s	Sand	1	17	5.4 MPa	1.3 MPa	0.25	2.2 m
	A-1c	Clay	1	19	2.9 MPa	1.2 MPa	0.40	0.44 m
	A-2s	Sand	1	20	5.4 MPa	1.1 MPa	0.20	1.2 m
	A-2c	Clay	1	21	4.2 MPa	0.95 MPa	0.23	0.26 m
	A-3s	Sand	1	18	10.5 MPa	3.2 MPa	0.31	2.1 m
	A-3c	Clay	1	29	6.1 MPa	1.7 MPa	0.27	1.1 m
	A-123s	Sand	3	55	7.1 MPa	3.2 MPa	0.45	4.8 m
	A-123c	Clay	3	69	4.6 MPa	1.9 MPa	0.41	1.5 m
B	B-1	Clay	1	48	2.8 MPa	0.74 MPa	0.27	0.21 m
	B-2	Clay	1	14	3.0 MPa	0.83 MPa	0.27	0.37 m
C	C-1	Sand	1	18	6.3 MPa	2.4 MPa	0.37	3.1 m
D	D-1	Clay	1	36	4.8 MPa	1.1 MPa	0.22	0.27 m

Note: N_{b} = number of core boring; n = sample size; $s\mu_{\text{quf}}$ = sample mean of core strength; $s\sigma_{\text{quf}}$ = sample standard deviation of core strength; sV_{quf} = sample coefficient of variation of core strength; $s\theta_{\text{quf}}$ = sample autocorrelation distance of core strength

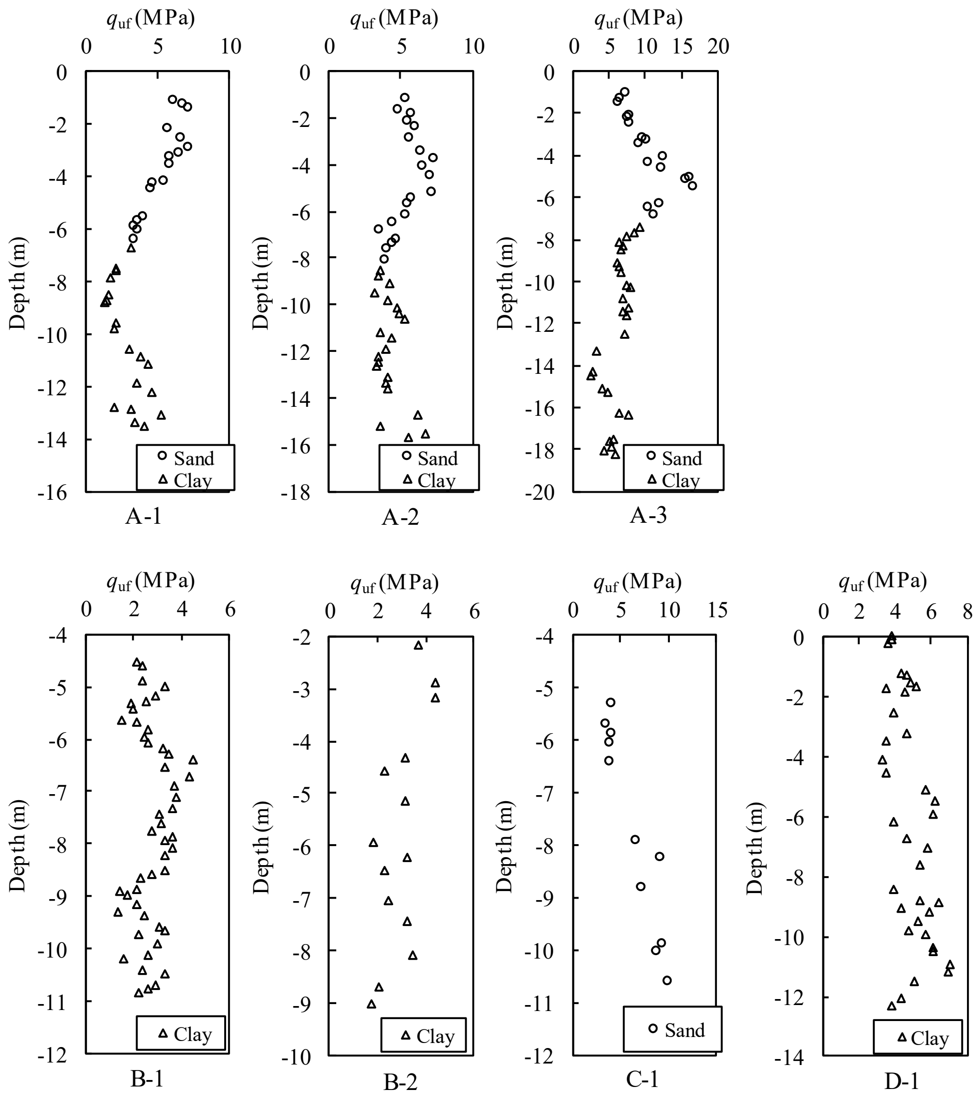

Figure 1. Distribution of core strength q_{uf}

ranges from 0.22 to 0.40. The sV_{quf} values of cement-treated sand dose not significantly differ from those of cement-treated clay. sV_{quf} for the three boring data (A-123s and A-123c) is larger than that for the single boring data in project A. This is because there is a large difference in the $s\mu_{quf}$ values of three cement-treated soil columns.

2.3 Probability Distribution

The probability distribution type of q_{uf} was investigated by the Kolmogoronv-Smirnov (K-S) test (Kanji 2006). The goodness fit of the normal distribution and log-normal distribution was examined for q_{uf}. Table 2 shows the K-S test results. The D statistic is defined as

$$D = \max|F(x) - S_n(x)| \tag{1}$$

where $F(x)$ is the target cumulative distribution function with the sample mean and variance and $S_n(x)$ is the sample cumulative distribution function. Since the D statistic is a maximum value of the difference between the sample and target cumulative distributions, the smaller value of D statistic indicates the better fit of the data. The D statistic in Table 2 shows that the normal distribution is better against the probability distribution of q_{uf} at some columns and the log-normal distribution is better against the probability distribution of q_{uf} at other columns. The *P*-value is the threshold value of the significant level. When a value is less than the *P*-value, the null hypothesis for the goodness fit will be accepted. Except for A-123s, the *P*-values of the both distributions are larger than 0.1, indicating that the both distributions are acceptable for the probability distribution of q_{uf} at the 10% significant level. The K-S test results suggest that the both distributions are possibly adopted for the probability distribution of q_{uf}.

Table 2. Kolmogoronv-Smirnov (K-S) test result

Project	Boring No.	Soil	Normal distribution		Log-normal distribution	
			D-value	*P*-value	*D*-value	*P*-value
A	A-1s	sand	0.148	0.799	0.182	0.563
	A-1c	clay	0.206	0.348	0.167	0.606
	A-2s	sand	0.096	0.984	0.135	0.810
	A-2c	clay	0.190	0.386	0.156	0.633
	A-3s	sand	0.136	0.852	0.137	0.844
	A-3c	clay	0.145	0.526	0.196	0.188
	A-123s	sand	0.196	0.025	0.112	0.459
	A-123c	clay	0.095	0.537	0.083	0.701
B	B-1	clay	0.076	0.946	0.096	0.769
	B-2	clay	0.139	0.917	0.184	0.651
C	C-1	sand	0.194	0.508	0.195	0.502
D	D-1	clay	0.115	0.726	0.112	0.756

2.4 Autocorrelation Distance

The autocorrelation distance θ_{quf} was adopted as a statistical parameter to represent the spatial correlation of the core strength q_{uf}. A stationary random field was assumed for the spatial variability of q_{uf}. The exponential function was adopted for the correlation coefficient $\rho(d)$ of q_{uf}. That function is expressed as

$$\rho(d) = \exp\left(-\frac{d}{\theta_{quf}}\right) \tag{2}$$

where *d* is the distance between the two points. The maximum-likelihood method was used to estimate θ_{quf}. A multivariate normal distribution was selected for the probability distribution of q_{uf}. The multivariate normal distribution is expressed as

$$p(q_{uf}) = \left(2\pi\sigma_{quf}^2\right)^{-\frac{n}{2}}|\mathbf{C}|^{-\frac{1}{2}}\exp\left\{-\frac{1}{2\sigma_{quf}^2}\left(\mathbf{q_{uf}} - \boldsymbol{\mu}_{\mathbf{quf}}\right)^{\mathrm{T}}\mathbf{C}^{-1}\left(\mathbf{q_{uf}} - \boldsymbol{\mu}_{\mathbf{quf}}\right)\right\} \tag{3}$$

$$\mathbf{q_{uf}} = \begin{bmatrix} q_{uf}(r_1) \\ \vdots \\ q_{uf}(r_n) \end{bmatrix},\quad \boldsymbol{\mu}_{\mathbf{uf}} = \begin{bmatrix} \mu_{quf} \\ \vdots \\ \mu_{quf} \end{bmatrix},\quad \mathbf{C} = \exp\left(-\frac{|\mathbf{r_i} - \mathbf{r_j}|}{\theta_{quf}}\right)$$

where **C** is the matrix of correlation coefficient, *n* is the number of q_{uf} and $\mathbf{r}_i$ is the space vector of the point *i*. The log-likelihood function $Lz(\theta_{quf})$ is defined as

$$Lz\left(\theta_{quf}\right) = -\frac{n}{2}\ln\left(2\pi\sigma_{quf}^2\right) - \frac{1}{2}\ln|\mathbf{C}| - \frac{1}{2\sigma_{quf}^2}\left(\mathbf{q_{uf}} - \boldsymbol{\mu}_{\mathbf{quf}}\right)^{\mathrm{T}}\mathbf{C}^{-1}\left(\mathbf{q_{uf}} - \boldsymbol{\mu}_{\mathbf{quf}}\right) \tag{4}$$

In maximum-likelihood method, the maximum-likelihood estimate of θ_{quf} is the θ_{quf} value at the maximum value of $Lz(\theta_{quf})$. Figure 2 shows that the calculated values of $Lz(\theta_{quf})$ for B-2 in project B. In this figure, the evaluated optimal value for θ_{quf} is 0.37 m. Table 1 shows that the evaluated optimal values for the sample autocorrelation distance $s\theta_{quf}$. The $s\theta_{quf}$ values calculated from the single column data ranges from 0.21 m to 2.2 m. Assuming no horizontal correlation of q_{uf} between the three borings, the $s\theta_{quf}$ values were calculated for A-123s and A-123c. The $s\theta_{quf}$ values for A-

123s and A-123c are larger than those for the single column data. It is inferred that the large difference in the $s\mu_{quf}$ values of the three core borings induces the large value of $s\theta_{quf}$.

The relationships between the sample coefficient of variation sV_{quf} and the sample autocorrelation distance $s\theta_{quf}$ is shown in Figure 3. It can be seen that $s\theta_{quf}$ of the cement-treated sand is larger than that of the cement-treated clay. This indicates that the spatial correlation of the cement-treated sand is stronger than that of the cement-treated clay. Figure 3 also shows that $s\theta_{quf}$ increases with sV_{quf}. This indicates that the strong spatial correlation possibly induces the large variability of q_{uf}.

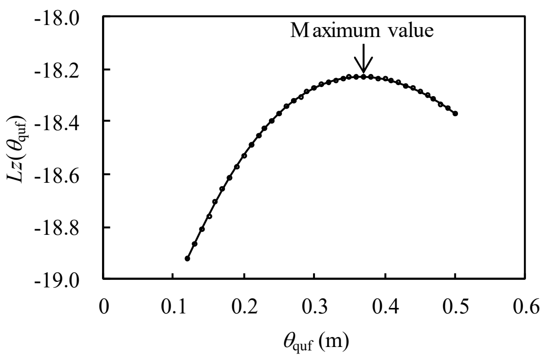

Figure 2. Log-likelihood function $Lz(\theta_{quf})$ for B−2 data

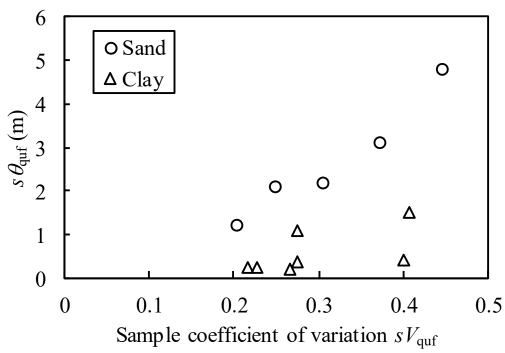

Figure 3. Sample autocorrelation distance $s\theta_{quf}$

3 STATISTICAL UNCERTAINTY IN STATISTICAL PARAMETERS

3.1 Evaluation Method of Statistical Uncertainty

Table 1 shows the sample statistical parameter values. The statistical uncertainty emerges in inferring the population statistical parameters from the sample values. The Bayesian inference approach proposed by Namikawa (2019) was used to evaluate the statistical uncertainty for the mean μ_{quf}, variance σ_{quf}^2, and autocorrelation distance θ_{quf}. In this approach, realizations of the population statistical parameters were generated using the MCMC method (Gamerman and Lopes 2006). The outline of the used approach is explained in the following section.

3.2 Bayesian Inference Approach

According to the Bayesian inference approach (Gelman et al. 2014), the population statistical parameters, μ_{quf}, σ_{quf}^2, and θ_{quf}, are defined as the joint probability distribution as

$$p(\mu_{quf}, \sigma_{quf}^2, \theta_{quf} | \mathbf{q_{uf}}) \propto p(\mathbf{q_{uf}} | \mu_{quf}, \sigma_{quf}^2, \theta_{quf}) p(\mu_{quf}) p(\sigma_{quf}^2) p(\theta_{quf}) \tag{5}$$

where $p(\mathbf{q_{uf}} | \mu_{quf}, \sigma_{quf}^2, \theta_{quf})$ is the probability distribution of q_{uf} under the given statistical parameters, and $p(\mu_{quf})$, $p(\sigma_{quf}^2)$, and $p(\theta_{quf})$ denote the prior distributions of μ_{quf}, σ_{quf}^2, and θ_{quf}. The posterior distribution defined by Equation (5) represents the probability distribution of the population statistical parameters inferred from the known core strength values.

The probability properties of μ_{quf}, σ_{quf}^2, and θ_{quf} are difficult to obtain from the joint probability distribution $p(\mu_{quf}, \sigma_{quf}^2, \theta_{quf} | \mathbf{q_{uf}})$. Here using the MCMC method, the realizations of μ_{quf}, σ_{quf}^2, and θ_{quf} are drawn sequentially from the conditional posterior distributions as

$$\begin{aligned} p(\mu_{quf} | \mathbf{q_{uf}}, \sigma_{quf}^2, \theta_{quf}) &\propto p(\mathbf{q_{uf}} | \mu_{quf}, \sigma_{quf}^2, \theta_{quf}) p(\mu_{quf}) \\ p(\sigma_{quf}^2 | \mathbf{q_{uf}}, \theta_{quf}, \mu_{quf}) &\propto p(\mathbf{q_{uf}} | \mu_{quf}, \sigma_{quf}^2, \theta_{quf}) p(\sigma_{quf}^2) \\ p(\theta_{quf} | \mathbf{q_{uf}}, \mu_{quf}, \sigma_{quf}^2,) &\propto p(\mathbf{q_{uf}} | \mu_{quf}, \sigma_{quf}^2, \theta_{quf}) p(\theta_{quf}) \end{aligned} \tag{6}$$

When the prior distribution is selected to be a natural conjugate distribution against the posterior distribution, the realizations of the parameters are obtained from the conditional posterior distribution directly. Since normal and inverse gamma distributions are natural conjugate distributions against the posterior distribution of μ_{quf} and σ_{quf}^2, these distributions are selected for $p(\mu_{quf})$ and $p(\sigma_{quf}^2)$. The truncated normal distribution is selected for $p(\theta_{quf})$. That distribution is not a natural conjugate distribution against the posterior distribution of θ_{quf}. In the MCMC procedure, the μ_{quf} and σ_{quf}^2 values were sampled by a Gibbs sampler and the θ_{quf} values were sampled by a Metropolis-Hastings algorithm. Namikawa (2019) has provided a detail description of the Bayesian inference approach adopted in this study.

The statistical parameters of the prior distribution were selected based on the past study (Namikawa 2019). The mean of μ_{quf} is 4 MPa and the standard deviation of μ_{quf} is 2 MPa. The mean of σ_{quf}^2 is 1 and the standard deviation of σ_{quf}^2 is 1. The mean of θ_{quf} is 1m and the standard deviation of θ_{quf} is 1m.

3.3 Analysis Results

An example of the population statistical parameter μ_{quf} drawn by the MCMC method is shown in Figure 4. The μ_{quf} values in this figure were inferred from the data of q_{uf} for B-1. The inferred population statistical parameters vary significantly, indicating that a large statistical uncertainty emerges in the evaluation of the population parameters.

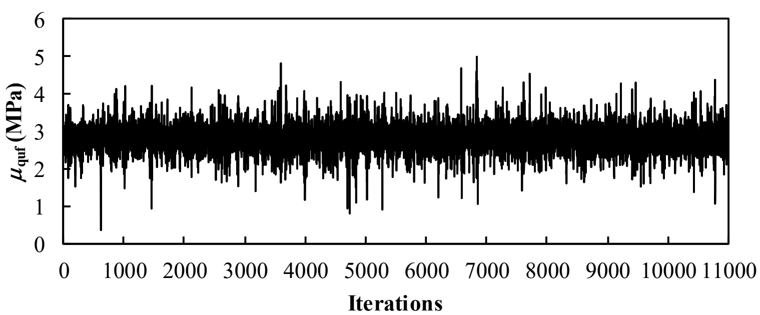

Figure 4. μ_{quf} drawn by the MCMC method for B-1

In all cases, 11000 realizations were drawn for the three parameters. The initial 1000 values were discarded in the calculation to reduce the effect of the starting values. The drawn values of μ_{quf}, σ_{quf}^2, and θ_{quf} are summarized as box plot graphs in Figure 5. The variation of the drawn values differs in each case. This indicates that the statistical uncertainty varies with the observed data. The spatial correlation and the sample size may affect the statistical uncertainty. The factors affecting the statistical uncertainty will be discussed in the following sections.

The mean values $\mu_{\mu quf}$, $\mu_{\sigma quf}$, and $\mu_{\theta quf}$ of μ_{quf}, σ_{quf}, and θ_{quf} drawn by the MCMC analysis were compared with the sample statistical parameters, $s\mu_{quf}$, $s\sigma_{quf}$, and $s\theta_{quf}$. The comparison of these values are shown in Figure 6. The mean values $\mu_{\mu quf}$ of μ_{quf} agree approximately with the sample mean values $s\mu_{quf}$ in the all cases except the A-3s case. In the MCMC analysis, the mean of the prior distribution of μ_{quf} is 4 MPa. Since $s\mu_{quf}$ for the A-3s case is much larger than that value, the prior distribution may affect the analysis result for the A-3s case. The mean values $\mu_{\theta quf}$ of θ_{quf} are lower than the sample autocorrelation distance $s\theta_{quf}$ for the A-123s and C-1 cases. In the MCMC analysis, the mean of the prior distribution of θ_{quf} is 1m. Since $s\theta_{quf}$ for these cases is much larger than that value, the prior distribution may affect the analysis result for these cases. Figure 6 indicates the mean values of the statistical parameters drawn in the MCMC analysis agree approximately with the sample statistical parameter values when the sample statistical parameter values do not differ significantly from the parameter values of the prior distribution.

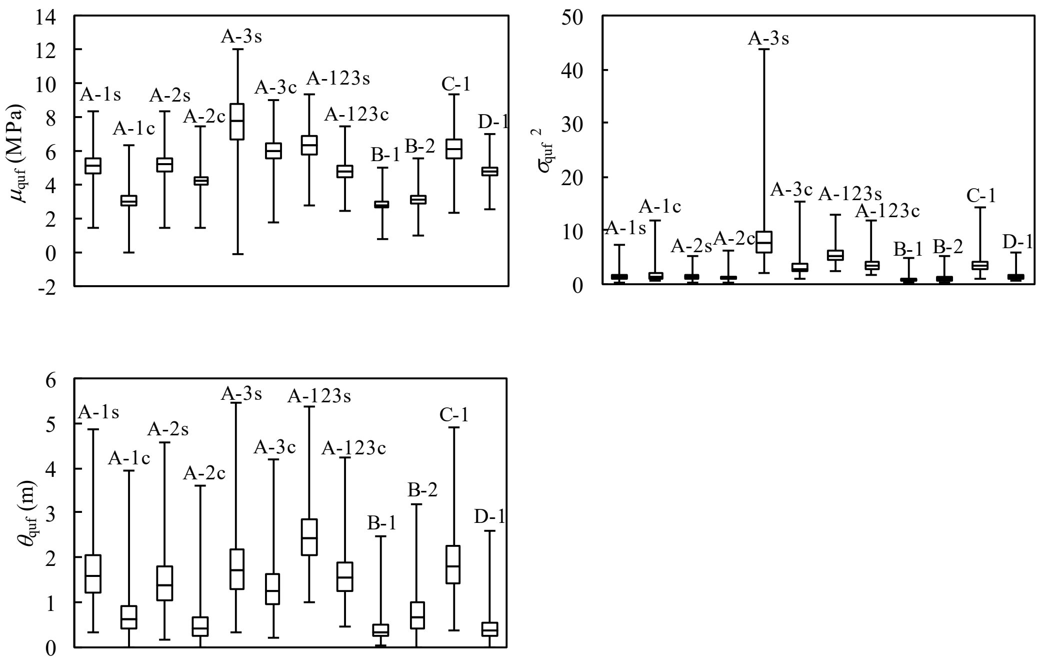

Figure 5. Box plot of mean $\mu_{quf,}$ variance σ_{quf}^2, and autocorrelation distance θ_{quf} values drawn by the MCMC method

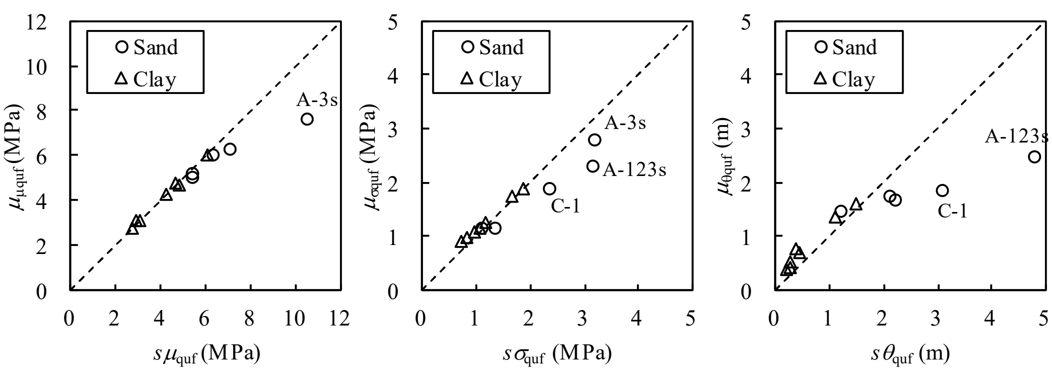

Figure 6. Comparison between sample statistical parameters, $s\mu_{quf}$, $s\sigma_{quf}$, $s\theta_{quf}$ and mean of population statistical parameters $\mu_{\mu quf}$, $\mu_{\sigma quf}$, $\mu_{\theta quf}$

3.4 Effective Sample Size

The sample size n affects the statistical uncertainty emerging in the evaluation of the population parameters. When the data has the spatial correlation, the effective sample size n_e that practically affects the statistical uncertainty depends on the sampling distance and the autocorrelation distance (Cressie 1993). The effective sample size n_e is defined as

$$n_{\mathrm{e}} = \frac{n^2}{\sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij}} \tag{7}$$

where C_{ij} is the (i, j) element of **C**.

Assuming that q_{uf} follows the normal distribution, an estimator of the standard deviation $e\sigma_{\mu quf}$ of μ_{quf} evaluated from the data with n_e is represented as

$$e\sigma_{\mu quf} = \frac{\sigma_{quf}}{\sqrt{n_e}} \tag{8}$$

Assuming that σ_{quf} corresponds to $s\sigma_{\text{quf}}$, Equation (4) is written as

$$e\sigma_{\mu\text{qu}} = \frac{s\sigma_{\text{quf}}}{\sqrt{n_e}} \tag{9}$$

Using Equations (7) and (9), the standard deviation of μ_{quf} that represents the statistical uncertainty included in the estimation of μ_{quf} can be simply estimated from the data of q_{uf}.

The standard deviation $\sigma_{\mu\text{quf}}$ of μ_{quf} was calculated from the MCMC analysis results. The comparison between $\sigma_{\mu\text{quf}}$ and $e\sigma_{\mu\text{quf}}$ is shown in Figure 7. $e\sigma_{\mu\text{quf}}$ corresponds reasonably to $\sigma_{\mu\text{quf}}$ in all cases except the A-3s, A-123s and C-1 cases. The mean inferred in the MCMC analysis is lower than the sample mean for A-3s and the autocorrelation distance inferred in the MCMC analysis is lower than the sample autocorrelation distance for the A-123s and C-1 cases (see Figure 6). It is inferred that the low value of the mean or the autocorrelation distance estimated in the MCMC analysis induces the low value of $\sigma_{\mu\text{quf}}$ for these cases. Figure 7 indicates that the statistical uncertainty of μ_{quf} can be simply estimated from the sample standard deviation and the effective sample size when the sample statistical parameters do not differ significantly from those of the past data which is used for the prior distribution.

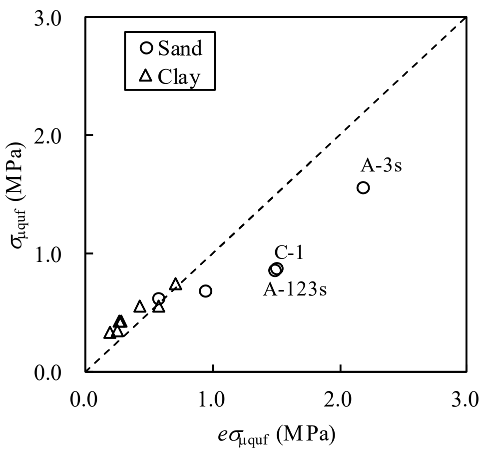

Figure 7. Comparison between estimator of standard deviation $e\sigma_{\mu\text{quf}}$ of μ_{quf} and standard deviation $\sigma_{\mu\text{quf}}$ of μ_{quf} drawn in MCMC analysis

4 CONCLUSIONS

The statistical analysis of the core strength observed in several deep cement mixing projects was conducted in this study. The mean μ_{quf}, standard deviation σ_{quf}, and autocorrelation distance θ_{quf} were evaluated as the statistical parameters of the unconfined compressive strength q_{uf}. The type of the probability distribution of q_{uf} was investigated by the Kolmogoronv-Smirnov (K-S) test. The K-S test results indicated that the normal and log-normal distributions can be adopted for the probability distribution of q_{uf}. Assuming that q_{uf} follows the normal distribution, the statistical uncertainty involving in the population parameters was evaluated using a Bayesian inference approach. MCMC method was adopted to calculate the realizations of the population statistical parameters. In the Bayesian inference analysis results, the statistical uncertainty emerges considerably when evaluating the population statistical parameters from the core strength data. The effective sample size n_{e} representing the number of the independent samples is a key factor for the amount of the statistical uncertainty. The standard deviation of μ_{quf} could be reasonably estimated from the sample standard deviation and n_{e}. The proposed method for evaluating the statistical uncertainty provides the useful information when assuring the strength of the deep cement mixing ground.

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