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Two-Dimensional Finite Element Analysis of Piled Raft Coefficient Settlement Ratio on Clays

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ABSTRACT Nowadays, designing a piled raft foundation is challenging because the behavior is three-dimensional. For some engineers, a three-dimensional analysis might not be affordable due to more costly than a two-dimensional analysis. In this study, 2D Finite element analysis – axisymmetric was used to study the piled raft foundation. The pile diameter and pile length were varied to investigate the relation between piled raft coefficient and load-settlement. In addition, the load transfer mechanism between the raft part and the pile part in clayey soils is also examined. The results show the longer the pile and the larger the diameter, the greater the load carried by the pile and the smaller the settlement. Increase in pile length by 5 m, resulting in a load transfer of 2% to 6% from a raft to pile, and reduced settlement by 2% to 3%. Furthermore, A 0.5 m increase in pile diameter results in an 8% to 25% load transfer from a raft to pile, and a 2% to 7% reduction in a settlement. The soil consistency affects the load distribution and settlement of the pile-raft foundation system. The higher the soil consistency, the smaller the amount transferred to the pile, and the higher the effectiveness of the pile in reducing the settlement that occurs.

KEYWORDS Piled rafts foundation; Finite element method; load transfer coefficient; settlement ratio

1 INTRODUCTION

There are several factors to consider when designing a foundation: load capacity, settlement, differential settlement, and torque. In current practices, both raft foundations and piled foundations are popular to be constructed. Typically, a shallow foundation, such as the raft foundation, is selected initially to reduce cost and construction time. If the existing foundation is insufficient, a piled foundation is used. To strengthen the foundation, the piles are frequently connected by their heads with a thick concrete slab. However, the design procedure of these typical pile group foundations does not take into account the contribution of the thick slab, which effectively acts as a raft foundation. As a result, the vast majority of pile groups are excessively designed and ineffective. This condition promoted the idea of incorporating both capabilities of pile groups and rafts which resulted in the Coupled Piled Raft Foundation (CPRF). Nevertheless, it has been proved that determining the contribution of both parts as a single unit is very challenging.

Currently, several methods have been developed to analyze the capacity of CPRFs. Burland et al (1977), Hain and Lee (1978), Katzenbach et al. (2000), and Poulos H. G (2001) developed a simplified method that includes theoretical hand calculated solutions for rafts and piles in the elastic continuum. This method is used to estimate the load carried by the piles. Then, there are approximate methods which include two approaches: strip on springs and plate on springs. Developed by Brown and Wiesner (1975), this method also does not require a computer but is limited to settlement calculation only. Recent design method developments have also been done by Mandolini et al (2013), and Kumar and Choudbury (2018). The latter proposed a new simplified prediction model for the bearing capacity and efficiency of the CPRFs at both the Ultimate Limit State (ULS) and the Serviceability Limit State (SLS).

In the last decade, numerical solutions using tools like the Finite Difference, Finite Element, and the Boundary Element Method such as Bernades et al (2019) also gained some popularity. Since most of the complexities of CPRFs mainly arise from their 3D geometry, these advanced methods allow more accurate prediction of the settlement of the piled raft. Maharaj and Gandhi (2004) used the Finite Element Method to develop a model and found that the load-carrying capacity of CPRFs increases as the raft size also increases. Also using the Finite Element Method, Patil et al (2020) created an estimation of CPRFs responses by using nonlinear soil and interface model. The computational analysis also brings machine learning in the analysis. Ghorbani & Niavol (2017) evaluated induced settlements of CPRFs using Neural Network Evolutionary Polynomial Regression.

The ability to model complex geometries also benefits in designing more sophisticated CPRFs with better geometrical effects such as Shi et al. (2019) and Ng et al. (2018). The rectangular piles are more convenient for transporting (Zhang et al, 2018). Tapered piles increase side resistance and alter the bucking behaviors (Lee et al, 2018). Lately, X-section cast-in-place concrete (XCC) piles have also been developed to increase bearing capacity by increasing the pile perimeter and altering the pile-soil interaction (Lv et al 2014, Lv et al 2018, Ding et al 2020).

Designing CPRFs is complex because there are many variables to consider such as pile configuration, pile dimensions, settlement, and load transfer mechanism. One of the foundation designing methods, the “t-z” analysis method, deals with load-transfer analysis. It calculates the vertical load displacements that happened on the pile head and compares it to the vertical load distribution along with the pile. In CPRFs, where the foundation is composed of two different parts, namely the raft part and the pile part, a coefficient is used to determine the contribution of each part in transferring load. These coefficients are highly beneficial as it allows a more simplified design process for CPRFs. Some studies have been conducted by Mascarucci et al (2015) and Alshenawy et al (2016) for the cases in sandy soil. The results show that piled raft coefficients are highly correlated to load-settlement behavior. These results also adhere to the correlation suggested by Katzenbach et al (2000). Meanwhile, the study of CPRFs in clay soils has been done by Mali and Singh (2018), and Susila et al (2019) which shows that increasing pile diameter will result in decreasing settlement ratio and increasing load-sharing ratio (i.e., Piled Raft Coefficients). Those studies were using the three-dimensional finite element method. For some engineers, the three-dimensional analysis might not be affordable due to being more costly than 2D analysis. In addition, it might also time consuming.

In this study, two-dimensional Finite element analysis – axisymmetric, simple yet well-known, was used to study the CPRFs. The pile diameter and pile length were varied to investigate the relation between the so-called piled raft coefficient and load-settlement to investigate the load transfer mechanism between the raft part and the pile part in clayey soils.

2 METHODOLOGY

2.1 Geometry, Model, and Mesh for Analysis

To simplify the model into two-dimensional geometry, an axisymmetric section is created as shown in Figure 1. Axisymmetric models are employed for circular objects with a (more or less) uniform radial cross-section and loading scheme around the central axis, where the deformation and stress state is considered to be the same in all radial directions (Plaxis manual, 2007).

The simulation consists of 9 models (see Table 1) whose dimensions are described Figure 1. For each model, the foundation will be evaluated in 3 types of clay soils with different consistency: soft, medium, and hard. This, resulting 27 variations of the simulations. And lastly, to create a load-transfer analysis, each variation will be subjected to 4 levels of loads: 25 kPa, 200 kPa, 500 kPa, and 1000 kPa. The boundary conditions at the left and right sides of the model are set to be a roller boundary, whereas, at the bottom, the boundary is set to be a fixed boundary. At the top, a distributed load is applied to the head of the raft foundation.

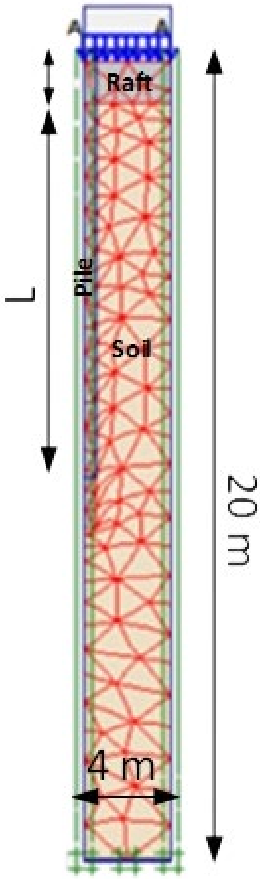


Figure 1 Finite Element Mesh and Model for Analysis

Table 1 The Variation of Pile Length and Diameter for Analysis

Diameter, d (m)	Pile Length, L (m)
0,5	10
	15
	20
1	10
	15
	20
1,5	10
	15
	20

The material model used to simulate the soil is the Elastic Perfectly Plastic model with the Mohr-Coulomb failure criterion. Meanwhile, to model the concrete of the raft foundation and the pile foundation, the linear elastic model is used. The Young's modulus and poison's ratio of concrete were assumed 21 GPa and 0.15, respectively. The Mohr-Coulomb failure criterion can be described as follows:

$$\tau = c + \sigma \tan \phi \tag{1}$$

where τ is the shear strength, σ is the normal stress, ϕ is the internal friction angle, and c is the cohesion.

In the analyses, total stress undrained analysis was chosen considering In this approach, soil stiffness and poison's ratio are modeled using an undrained Young's modulus (E_u) and undrained Poisson's ratio (υ_u). Meanwhile, the soil shear strength is expressed with an undrained shear strength ($c = S_u$) and $\phi = \phi u = 0$. Later, an automatic mesh generator is utilized to discretize the model with coarseness settings set to be "medium" as shown in Figure 1**.** A 15-node triangular element was applied in this analysis. It uses fourth-order interpolation for displacements, and the numerical integration requires twelve Gauss points (stress points). According to the Plaxis 2D manual, a 15-node triangular element is suggested to be applied in axisymmetric simulation. This element could yield high quality and accurate stress (Sloan, S. W, 1981, Sloan & Randolph, 1982, and Nagtegaal et al, 1974). The mesh created appear denser in the foundation and sparser in the soil. The total elements generated in this model are 223 elements.

The input parameter of each soil type is presented in Table 2. Because of the nature of clay soil, varying the consistency translate to varying cohesion and Young's modulus. The parameter used in this study follows Lim A (2011) with a corresponding adhesion factor or interface ratio based on Kulhawy (1991) for bored piles.

Table 2 The Input Parameter of Soil and Structure Elements for Analysis

Parameter			Clay Soil			Concrete
			Soft	*Medium*	*Stiff*	
Constitutive Model			Mohr Coulomb	Mohr Coulomb	Mohr Coulomb	*Linear Elastic*
Material Type			*Drained*	*Drained*	*Drained*	*Drained*
Saturated Unit Weight of soil	γ_{sat}	kN/m^3	20	20	20	24
Undrained Young's modulus	E_u	kPa	5000	17500	75000	21000000
Poisson ratio	υ_u	-	0.495	0.495	0.495	0.15
Internal friction angle	ϕ	°	0	0	0	-
Undrained shear strength	$c = S_u$	kPa	10	35	150	-
Interface coefficient	R_{inter}	-	1	0.93	0.39	-

2.2 Calculation Phases

The typical calculation phases used for analysis are as follows:

First, generating initial stresses in soil elements by using the K_0 – procedure. The coefficient of at-rest pressure is followed by the *Jaky* formula as shown in equation 2.

$$K_0 = 1 - \sin \phi \tag{2}$$

Second, activating the pile and raft foundations by changing the cluster becomes a concrete model. Third, resetting the displacement to zero. This action is for eliminating the pile-raft installation effect. In other words, the pile-raft foundation is wished in place. This approach has been well adopted by other researchers to model concrete structures in the soil (Lim et al, 2022). Fourth, the distributed load was applied on the top of the pile-raft foundation.

2.3 Settlement Ratios and Piled Raft Coefficient

With the CPRFs paradigm, the use of piles is to reduce settlement and assist load bearing. The evaluation of the effectiveness of using piles in reducing settlements could be assessed with the settlement ratio (β). The definition of settlement ratio is a ratio of the total settlement of the pile-raft system to the settlement of the raft-only system, as shown in equation (3).

$$\beta = \frac{U_{pile-raft}}{U_{raft}} \tag{3}$$

Where Upile-raft is the total settlement of the pile-raft system, and U_{raft} is a total settlement of the raft-only system In CPRFs, the desirable β value would be close to zero. This means that the pile foundation could reduce settlement significantly. In this analysis, the calculated settlement is limited to the elastic settlement.

The piled raft coefficients are defined as the changes in the load-settlement response of the pile group when the raft is being rested on the soil. This coefficient can be evaluated as the ratio between the total resistance held by the piles to the total resistance of both the pile and the rafts, as shown in equation (4).

$$\alpha_{pp} = \frac{\sum R_{pile}}{R_{tot}} \tag{4}$$

With such a definition, it is implied that a coefficient of 0 (zero) means the CPRFs are behaving like shallow foundations. Meanwhile, a coefficient of 1 (one) means the CPRFs are behaving totally like piled foundations without any contribution from the raft part. In general, all CPRFs have coefficients in the range of 0 to 1. It is rare to encounter a case where the raft does not give any contribution. These cases could only be seen when there is no contact pressure between the raft and the soil. The typical settlement of the analysis is shown in Figure 2. The value was obtained at the top of the pile-raft foundation. At this location, the settlement is uniform.

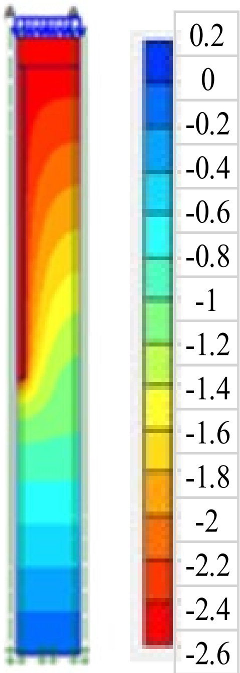
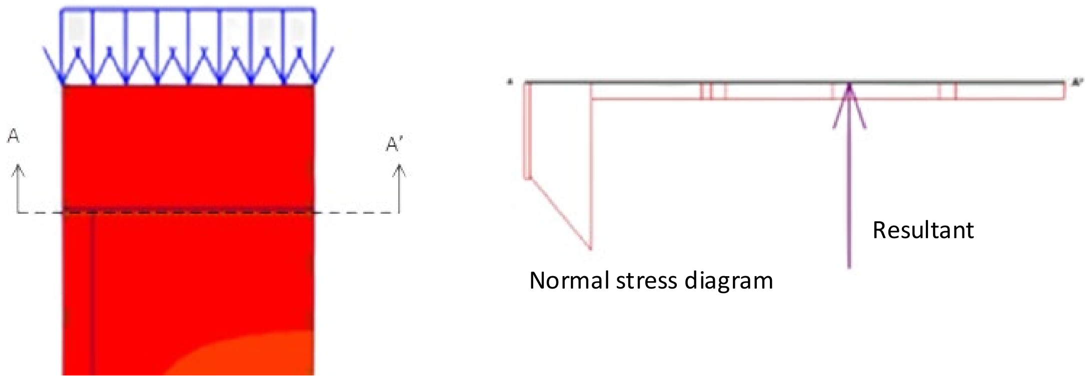


Figure 2 Typical Output for Vertical Displacements

Moreover, to obtain the load acting on the raft, a cross-section A-A' was made, as depicted in Figure 3. Cross-section A-A' was located about 5 mm to 10 mm below the raft. It shows the normal stress acting below on the raft. By multiplying the normal stress with the section area of the raft, the load bear by the raft could be obtained. For getting the load bear by the pile, it is simply subtracting the total load from the load bear by the raft. Hence, app and b could be summarized.



(a) (b)

Figure 3 Typical Output for Normal Stress. (a) Location of the Cross-Section A-A' and (b) Normal Stress Diagram for Cross-Section A-A'

3 RESULTS AND DISCUSSION

3.1. *Piled Raft Coefficients*

Figure 4(a) provided the findings of the piled raft coefficient for soft clay. When the foundation is subjected to a load of up to 200 kPa, the overall findings reveal an increase in coefficient. This signifies that, up to this point, the piles have supported the majority of the load. In soft clay, however, applying more loads reduces the coefficient. As a result, it is shown that the raft component begins to carry loads only after some deformation occurs.

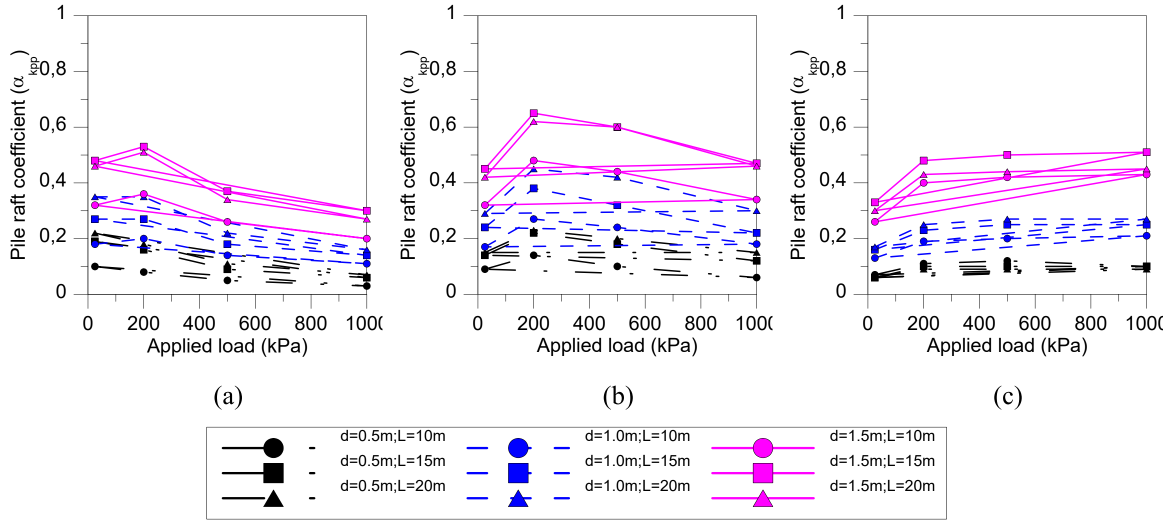


Figure 4 Results of Pile-Raft Coefficient for (a) Soft Clay, (b) Medium Clay, and (c) Hard Clay

The amount at which the coefficient decreased appears to be proportional to the soil's consistency. After reaching a specific maximum point in soft clay soils, the coefficients declined dramatically as the load increased. This pattern becomes more pronounced as the pile diameter and length increase. However, in situations where the diameter of the piles is small enough, the raft has played a larger part in supporting the weight from the start of the loading process. As a result, the graph shows no improvement in the coefficient, but rather a progressive decrease.

Figure 4(b) depicts the result of the simulations performed in medium clay. At the beginning of loading, the pile-raft foundation with a pile diameter of 0.5 m has a α_{pp} value ranging from 0.09 to 0.15. For a pile diameter of 1m, the value of α_{pp} ranges from 0.17 to 0.29. And for the 1.5 m diameter pile, the value of app ranges from 0.32 to 0.42. It can be seen that the larger the diameter of the pile, the higher the value of α_{pp}, which means that the load borne by the pile is getting greater. For variations in pile length, the pile-raft foundation with a pile length of 10m, has a value of α_{pp} which ranges from 0.09 to 0.32. For a pile length of 15 m, the value of α_{pp} ranges from 0.14 to 0.45. And for the 20 m pole length, the value of α_{pp} ranges from 0.15 to 0.42. Although the load carried by the raft is still greater than the load carried by the piles, it can be seen that the correlation between the diameter and the length of the piles and the load carried by the piles is directly proportional.

The modeling results in hard clay soils were shown in Figure 4(c). In contrast to prior observations, the coefficient in hard clay soils appears to steadily rise rather than decrease. This indicates that in hard clay soils, piles contribute more to load resistance than in weaker clay soils. The magnitude of the coefficients varies very little after the 200 kPa load.

3.2. Settlement Ratio

Figure 5 (a) shows the settlement ratio for soft clay. At the beginning of loading, the pile-raft foundation with a pile diameter of 0.5m has a value ranging from 0.86 to 0.97. For a pile diameter of 1 m, the value ranges from 0.76 to 0.87. And for the 1.5 m diameter pole, the value ranges from 0.68 to 0.79. It can be seen that the larger the diameter, the lower the settlement ratio which means that the pile has succeeded in minimizing the settlement that occurs in the pile-raft foundation system. Such behavior can be observed throughout the loading stages, and also for medium clay and hard clay in Figure 5(b) and Figure 5(c).

For variations in pile length in soft clay illustrated in Figure 5(a), the pile-raft foundation with a pile length of 10 m has values ranging from 0.79 to 0.97. For 15 m pile length, the value ranges from 0.73 to 0.89. And for the 20 m pile length, the value ranges from 0.68 to 0.86. It can be seen that the longer length of the pile, the lower the settlement ratio which means that the pile has succeeded in minimizing the settlement that occurs in the pile-raft foundation system. Such behavior can be observed also in medium clay and hard clay.

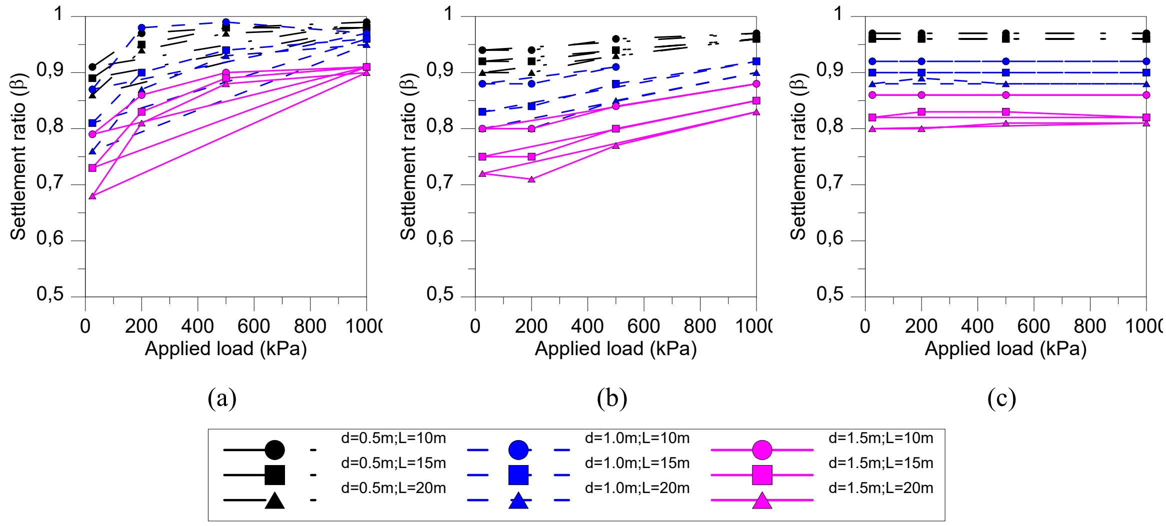


Figure 5 Results of The Settlement Ratio for (a) Soft Clay, (b) Medium Clay, and (c) Hard Clay

As the pile raft foundation was embedded in soft clay and medium clay, the settlement ratio was changed depending on the applied load. The small, applied load yielded a smaller settlement ratio. But this trend was not found in the hard clay. In the hard soil, the settlement ratio is relatively larger than in the soft and medium clays. It indicates the contribution of raft foundation is more significant in the hard soil than in the medium and clay soils.

3.3. Relationship Between Settlement Ratio and Piled Raft Coefficients

Figure 6 presented a plot of piled raft coefficients to its corresponding settlement ratio. The result is, as can be seen, conform to the suggestion made by Katzenbach et al. (2000).

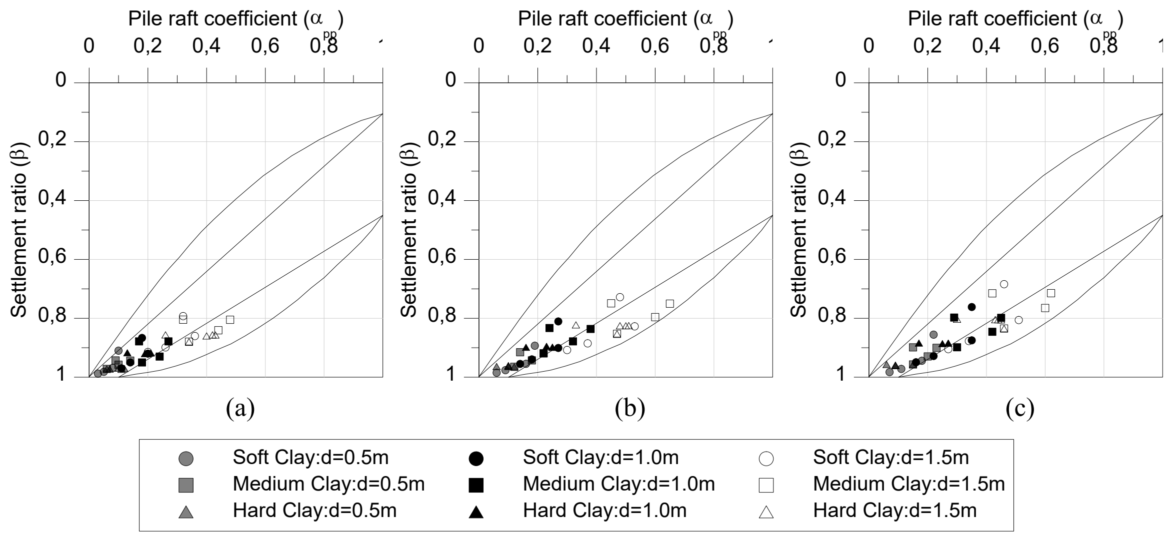


Figure 6 Corelation Between Piled Raft Coefficient and Settlement Ratio for (a) L = 10m, (b) L = 15m, and (c) L = 20m

This result shows that the variation created in this study is following Katzenbach's research. From the graph, it could be deduced that the consistency of the soil does not affect the distribution of the points in this correlation. On the other hand, the diameter of the piles does affect the distribution. Larger piles yield higher piled raft correlations, thus subsequently lower settlement ratio. Moreover, it could also be concluded that 2D finite element analysis with axisymmetric is suitable to model pile-raft foundation. It is relatively faster and could be useful for analyzing pile-raft foundations.

4 CONCLUSIONS

The conclusions that can be drawn from the investigation study of the load distribution mechanism on the pile-raft foundation using the finite element method are as follows:

1. The length of the pile affects the distribution of the load and the settlement of the pile-raft foundation system. The longer the pile, the greater the load carried by the pile and the smaller the settlement. Increase in pile length by 5 m, resulting in a load transfer of 2% to 6% from a raft to pile, and reduced settlement by 2% to 3%.
2. The diameter of the pile affects the distribution of the load and the settlement of the pile-raft foundation system. The larger the diameter of the pile, the greater the load carried by the pile, and the smaller the settlement that occurs. A 0.5 m increase in pile diameter results in an 8% to 25% load transfer from a raft to pile, and a 2% to 7% reduction in a settlement.
3. Soil consistency affects the load distribution and settlement of the pile-raft foundation system. The higher the soil consistency, the smaller the amount transferred to the pile, and the higher the effectiveness of the pile in reducing the settlement that occurs.
4. The load distribution mechanism that occurs in the pile-raft foundation system is that the greater the load applied to the pile-raft foundation system, the greater the load transmitted to the raft, this phenomenon can be observed from the α_{pp} graph on soft and medium clay soils, where the value of α_{pp} decreases as the load increases. In the hard clay soils, the value of α_{pp} is higher although not so significant, which means that the load is transferred to the pile when the load increases.

DISCLAIMER

The authors declare no conflict of interest.

AVAILABILITY OF DATA AND MATERIALS

All data are available from the author.

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